The instant when the droplet touched the surface was recorded in each experiment. Then from a certain instant of time when $r_1 > R$ (r_1 is distance from the center to the first crest) through a time interval of $\Delta t = 10^{-4}$ sec distance r, and the distance between crests were determined: $\lambda_1 = |r_1 - r_2|$, $\lambda_2 = |r_2 - r_3|$.

Represented in by curves 1-3 in Fig. 2 are the time dependences obtained $r_1(t)$, $\lambda_1(t)$, and $\lambda_2(t)$. Each curve is a summary of experimental data for a wave system formed by droplets with different parameters. Time t = 0 is the instant of droplet contact with the surface. Broken lines refer to theoretical dependences which are easily obtained from (2) assuming that $\sigma = 0.72$ N/m, $\rho = 1000$ kg/m^3, m = 0, 1, 2 for r_1, r_2, r_3 respectively: $r_1 = 8.9\ t^{2/3}$, $\lambda_1 = r_1 - r_2 = 6.3\,t^{2/3}$, $\lambda_2 = r_2 - r_3 = 3.2\ t^{2/3}$. The good agreement of experimental and theoretical data can be seen from Fig. 2.

LITERATURE CITED

1. H. Lamb, Hydrodynamics, Dover, New York (1932).
2. J. J. Stoker, Water Waves, Wiley Interscience, New York (1957).
3. G. B. Whitham, Linear and Nonlinear Waves, Wiley-Interscience, New York (1974).
4. B. Le Mehaute, "Gravity-capillary rings generated by water drops," J. Fluid. Mech., 197 (1988).
5. G. D. Crapper, Introduction to Water Waves, Wiley, New York (1984).
6. A. V. Kolpakov, V. A. Salov, and E. M. Dmitrieva, "Procedure and device for studying charging of a droplet during impact," Fiz. Aérodispersnykh Sistem, No. 32 (1989).
7. W. R. Wolf, "Study of a vibrating reed in the production of small droplets and solid particle of uniform size," Rev. Sci. Instrum., 32, No. 10 (1961).
8. N. N. Kochurova and A. I. Rusanov, "Properties of a non-equilibrium water surface and aqueous solutions," Kolloid. Zh., 43, No. 1 (1981).

INFLUENCE OF CAPILLARY FORCES ON THE NONSTATIONARY FALL OF A DROP IN AN UNBOUNDED FLUID

L. K. Antanovskii

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A large number of works are devoted to the problem of the dynamics of a viscous fluid drop (see [1-13]). At present the problem of the motion of a drop under the action of surface tension forces is of particular interest. These forces depend in essence on the temperature and concentration of surface-active substances (SAS) at the boundary separating the fluids. This interest is primarily determined by the requirements of chemical technology [3, 7] and the development of space studies [5, 14], where one must be able to predict the behavior of fluids in weak force fields and in conditions of weightlessness. In this work, an explicit solution is constructed for the linear problem of drop motion in an unbounded fluid with SAS present. The number of SAS is arbitrary (to a first approximation, chemical reactions are not taken into account).

Let a drop of viscous, incompressible fluid be located in another fluid with low SAS concentrations in solution, and let the drop begin slow motion under the action of variable acceleration of a volume force $\mathbf{g}(t)$ (t is the time). As a result of dilatation of the separation boundary Γ, the thermodynamic equilibrium of SAS is displaced in the volume and at the surface. This leads to an additional capillary force, which retards the motion of the drop. In other words, the Le Chatelier principle holds: the external action on a system in a stable state of thermodynamic equilibrium produces a reaction in the system which reduces the effect of the external action. The latter important property of reactivity of capillary forces is directly related to the fundamental principles of thermodynamics. Consistent application of these principles makes it possible to write the equations of thermal

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diffusion of SAS in the volume and at the separation boundary in symmetric form [12, 15], and to reasonably simply find a solution to the linearized problem.

We assume that the weak concentration of SAS molecules and the temperature do not affect the physical properties of the drop and the surrounding fluid, but that they do change the coefficient of surface tension σ. (The density ρ and dynamic viscosity coefficient $\mu = \rho\nu$ are piecewise constant on the surface of separation Γ). We write the Navier–Stokes equations in a noninertial coordinate system $(\mathbf{x}, t)$, fixed to the center of mass of the drop, whose position in the observer's inertial coordinate system is given by the vector $\mathbf{z}(t)$. Assuming that at the initial instant of time, the spherical drop is at rest, we obtain the problem with a free boundary [16]

$$\rho d\mathbf{v}/dt = \rho \mathbf{g} - \rho d^2\mathbf{z}/dt^2 - \operatorname{div} \mathbf{P},\ \operatorname{div} \mathbf{v} = 0 \text{ outside of } \Gamma; \tag{1}$$

$$[\mathbf{P}\cdot\mathbf{n}] = \operatorname{div}_\Gamma(\sigma\nabla_\Gamma \mathbf{x}),\ d\mathbf{x}/dt = \mathbf{v},\ [\mathbf{v}] = 0 \text{ on } \Gamma \tag{2}$$

$$\mathbf{v} \to -d\mathbf{z}/dt,\ p - \mathbf{x}\cdot\nabla p \to 0 \text{ for } |\mathbf{x}| \to \infty; \tag{3}$$

$$\mathbf{v} = 0,\ \mathbf{z} = 0,\ d\mathbf{z}/dt = 0,\ \Gamma = \{|\mathbf{x}| = a\} \text{ for } t = 0. \tag{4}$$

Here $\mathbf{v}$ is the velocity; p the pressure; $\mathbf{P} = p\mathbf{I} - 2\mu\mathbf{D}$ is the pressure tensor ($\mathbf{D}$ is the deformation rate tensor); $\mathbf{n}$ is the unit normal vector; a is the mean radius of the drop; the lower index Γ denotes surface differential operators; the square brackets denote the operation of computing the jump in the function during transit of the surface Γ along $\mathbf{n}$, namely: $[F] = F_+ - F_-$, $F_\pm(\mathbf{x}) = \lim\limits_{\delta\to\pm0} F(\mathbf{x} + \delta\mathbf{n}(\mathbf{x}))$.

Note that $\operatorname{div}_\Gamma(\sigma\nabla_\Gamma\mathbf{x}) = \nabla_\Gamma\sigma + \sigma k\mathbf{n}$ (k is the sum of the principal curvatures of Γ). Therefore, for variable σ, the capillary forces not only induce a jump in the pressure at Γ, but a jump in the tangential stress as well, which can substantially affect the drop dynamics [3]. The quantity σ is a thermodynamic parameter and consequently is determined by the thermodynamic state of the medium.

Let e be the specific density and ε the surface density of internal energy; c the volume and γ the surface concentration of SAS molecules; and $\mathbf{q}$ and $\mathbf{j}$ be the vector fluxes of heat and SAS in the fluid. Then the differential laws of conservation of energy and SAS mass are written in the form

$$\rho de/dt = 2\mu|\mathbf{D}|^2 - \operatorname{div}\mathbf{q} \text{ outside of } \Gamma; \tag{5}$$

$$\rho dc/dt = -\operatorname{div}\mathbf{j} \text{ outside of } \Gamma; \tag{6}$$

$$d\varepsilon/dt + \varepsilon \operatorname{div}_\Gamma \mathbf{v} = \sigma \operatorname{div}_\Gamma \mathbf{v} - [\mathbf{q}\cdot\mathbf{n}] \text{ on } \Gamma; \tag{7}$$

$$d\gamma/dt + \gamma \operatorname{div}_\Gamma \mathbf{v} = -[\mathbf{j}\cdot\mathbf{n}] \text{ on } \Gamma. \tag{8}$$

We choose the absolute temperature θ and the SAS chemical potential ζ as the independent thermodynamic parameters. Then the extensive variables can be expressed as partial derivatives of the thermodynamic potential $f(\theta, \zeta)$ in the volume

$$s = -\partial f/\partial\theta,\ c = -\partial f/\partial\zeta,\ e = f - \theta\partial f/\partial\theta - \zeta\partial f/\partial\zeta \tag{9}$$

and by the coefficient of surface tension $\sigma(\theta, \zeta)$ on the surface Γ [17]

$$\eta = -\partial\sigma/\partial\theta,\ \gamma = -\partial\sigma/\partial\zeta,\ \varepsilon = \sigma - \theta\partial\sigma/\partial\theta - \zeta\partial\sigma/\partial\zeta \tag{10}$$

(s, η are the specific and surface entropy densities). In view of the concavity of $f(\theta, \zeta)$ and $\sigma(\theta, \zeta)$, the matrices of their second derivatives are negative definite [15]; the Onsager relations lead to

$$-\mathbf{h} = K^{11}\nabla\theta + K^{12}\nabla\zeta,\ -\mathbf{j} = K^{21}\nabla\theta + K^{22}\nabla\zeta \tag{11}$$

with symmetric positive definite matrices $\{K^{ij}\}$ ($\mathbf{h}$ is the entropy flux vector).

Using (9)-(11), Eqs. (5)-(8) in intensive variables $\{\theta, \zeta\}$ can be written in symmetric form [15]

$$\rho d(-\partial f/\partial\theta)/dt = \Phi - \operatorname{div}\mathbf{h} \text{ outside of } \Gamma; \tag{12}$$

$$\rho d(-\partial f/\partial\zeta)/dt = -\operatorname{div}\mathbf{j} \text{ outside of } \Gamma; \tag{13}$$

$$d(-\partial\sigma/\partial\theta)/dt = (\partial\sigma/\partial\theta)\operatorname{div}_\Gamma \mathbf{v} - [\mathbf{q}\cdot\mathbf{n}] \text{ on } \Gamma; \tag{14}$$

$$d(-\partial\sigma/\partial\zeta)/dt = (\partial\sigma/\partial\zeta)\operatorname{div}_\Gamma \mathbf{v} - [\mathbf{j}\cdot\mathbf{n}] \text{ on } \Gamma; \tag{15}$$

$$[\theta] = [\zeta] = 0 \text{ on } \Gamma \tag{16}$$

($\Phi = (2\mu|\mathbb{D}|^2 - \mathbf{h}\cdot\nabla\theta - \mathbf{j}\cdot\nabla\zeta)/\theta$ is the dissipation function). Equation (15) comes from the principle of local thermodynamic equilibrium. Equations (1)-(4), (11)-(16) together with initial data for θ and ζ completely determine the dynamics of the drop in the presence of SAS.

Relations (11)-(16) are generalized in a natural way for the case of many SAS, taking into account their flux along Γ. Let $\tau = \{\tau_i\}$ be the set of intensive thermodynamic parameters (in the previous case, $\tau_1 = \theta$, $\tau_2 = \zeta$), and $f(\tau)$, $\sigma(\tau)$ the specific and surface densities of the Gibbs potential, which are concave functions of τ. By introducing the notation

$$B^{ij}(\tau) = -\frac{\partial^2 f(\tau)}{\partial\tau_i\partial\tau_j}, \quad \beta^{ij}(\tau) = -\frac{\partial^2\sigma(\tau)}{\partial\tau_i\partial\tau_j}, \quad \sigma^i(\tau) = \frac{\partial\sigma(\tau)}{\partial\tau_i},$$

we obtain the system

$$\rho B^{ij} d\tau_j/dt = \operatorname{div}(K^{ij}\nabla\tau_j) + \Phi^i \text{ outside of } \Gamma; \tag{17}$$

$$\beta^{ij} d\tau_j/dt = \sigma^i \operatorname{div}_\Gamma \mathbf{v} + [K^{ij}\partial\tau_j/\partial n] + \operatorname{div}_\Gamma(\varkappa^{ij}\nabla_\Gamma\tau_j) + \Phi^i_\Gamma, \quad [\tau_i] = 0 \text{ on } \Gamma. \tag{18}$$

Here repeated indices are summed, and the symmetric matrices $\{B^{ij}\}$, $\{K^{ij}\}$, $\{\beta^{ij}\}$, and $\{\kappa^{ij}\}$ are positive definite. The dissipation functions Φ^i, $\Phi^i{}_\Gamma$, which can account for chemical reactions, for simplicity will be assumed to be quantities of second order smallness compared to the perturbations $\mathbf{v}$, τ_i on the state of rest. Therefore their exact form is immaterial.

We introduce the dimensionless parameter $\delta = \max\{|[\rho]g|a^3/\mu\nu\}$, which we assume is much less than unity, which is a priori ensured for a sufficiently small drop. For $\delta = 0$ there is an exact solution to (1)-(4), (17), (18)

$$\Gamma = \{|\mathbf{x}| = a\}, \quad \mathbf{v} = \mathbf{0}, \quad p = \begin{cases} 2\sigma/a & \text{for } |\mathbf{x}| < a, \\ 0 & \text{for } |\mathbf{x}| > a, \end{cases} \quad \tau_i = \text{const.}$$

We linearize the problem in this parameter, keeping the previous notation for the perturbations $\mathbf{v}$, p, τ_i. We find as a result

$$\partial\mathbf{v}/\partial t = \mathbf{g} - d^2\mathbf{z}/dt^2 - \nabla p/\rho + \nu\Delta\mathbf{v}, \ \operatorname{div}\mathbf{v} = 0 \text{ outside of } \Gamma; \tag{19}$$

$$[\mathbf{P}\cdot\mathbf{n}] = \operatorname{div}_\Gamma(\sigma^i\tau_i\nabla_\Gamma\mathbf{x}), \ [\mathbf{v}] = \mathbf{0}, \ \mathbf{v}\cdot\mathbf{n} = 0 \text{ on } \Gamma; \tag{20}$$

$$\mathbf{v} \to -d\mathbf{z}/dt, \ p - \mathbf{x}\cdot\nabla p \to 0 \text{ for } |\mathbf{x}| \to \infty; \tag{21}$$

$$\rho B^{ij}\partial\tau_j/\partial t = K^{ij}\Delta\tau_j \text{ outside of } \Gamma; \tag{22}$$

$$\beta^{ij}\partial\tau_j/\partial t = \sigma^i \operatorname{div}_\Gamma \mathbf{v} + [K^{ij}\partial\tau_j/\partial n] + \varkappa^{ij}\Delta_\Gamma\tau_j, \quad [\tau_i] = 0 \text{ on } \Gamma; \tag{23}$$

$$\mathbf{v} = \mathbf{0}, \ \mathbf{z} = \mathbf{0}, \ d\mathbf{z}/dt = \mathbf{0}, \ \tau_i = 0 \text{ for } t = 0. \tag{24}$$

The coefficients of $\{B^{ij}\}$ and $\{K^{ij}\}$ (and ρ, μ, and ν as well) are piecewise constant with a surface of discontinuity Γ, and $\{\beta^{ij}\}$, $\{\kappa^{ij}\}$, $\{\sigma^{ij}\}$ are constants, since they are computed for a state of equilibrium. In addition, the fact that problem (19)-(24) admits an exact solution for spherical free boundaries [11] is considered.

We temporarily assume that $\mathbf{g} = g(t)\mathbf{e}$ ($\mathbf{e}$ is a fixed unit vector) and we introduce a spherical coordinate system (r, ϑ, ϕ) such that $r = |\mathbf{x}|$, and $\cos\vartheta = \mathbf{e}\cdot\mathbf{x}/|\mathbf{x}|$. Then it is possible to seek an axisymmetric solution of the form

$$v_r = -2r^{-1}\psi(r, t)\cos\vartheta, \ v_\vartheta = r^{-1}\{\partial(r\psi(r, t))/\partial r\}\sin\vartheta,$$

$$\tau_i = \chi_i(r, t)\cos\vartheta, \ \mathbf{z} = z(t)\mathbf{e}$$

($\psi(r, t)$, $\chi_i(r, t)$, $z(t)$ are the unknown functions). After a Laplace transform in time, considering the homogeneity of the initial data, we have the following problem

$$S^2(\nu S^2\psi^* - \lambda\psi^*) = 0 \text{ for } r \neq a; \tag{25}$$

$$\psi^* = 0, \ [\psi^*] = 0, \ [\partial\psi^*/\partial r] = 0, \tag{26}$$

$$[\mu\partial^2\psi^*/\partial r^2] = \sigma^i\overset{*}{\chi}_i/a \quad \text{for} \quad r = a; \tag{27}$$

$$\partial\psi^*/\partial r, \ \psi^*/r \to \lambda z^*/2 \text{ for } r \to \infty;$$

$$\rho\lambda B^{ij}\chi_j^* = K^{ij}S^2\chi_j^* \quad \text{for} \quad r \neq a; \tag{28}$$

$$\lambda\beta^{ij}\chi_j^* = 2\{\sigma^i\partial\psi^*/\partial r + \varkappa^{ij}\chi_j^*/a\}/a + [K^{ij}\partial\chi_j^*/\partial r], \quad [\chi_i^*] = 0 \quad \text{for} \quad r = a; \tag{29}$$

$$a[\rho](g^* - \lambda^2 z^*) = [\partial\{r(\mu S^2\psi^* - \rho\lambda\psi^*)\}/\partial r] - 2\sigma^i\chi_i^*/a \quad \text{for} \quad r = a. \tag{30}$$

Here $S^2f = r^{-2}\{\partial(r^2\partial f/\partial r)/\partial r - 2f\}$: the normal $\mathbf{n} = \mathbf{x}/a$ completely determines the sign of the jump; λ is the conjugate variable of the Laplace transform.

It is evident that the general solution to Eqs. (25) and (27) can be represented as [11]

$$\psi^*(r, \lambda) = \begin{cases} \lambda z^*(\lambda)r/2 + A_1(\lambda)/r^2 + A_2(\lambda)H\left(r\sqrt{\lambda/\nu_+}\right) & \text{for} \quad r > a, \\ A_3(\lambda)r + A_4(\lambda)H\left(r\sqrt{\lambda/\nu_-}\right) & \text{for} \quad r < a, \end{cases}$$

where the function $H(\xi) = (e^{-\xi}/\xi)'$ for $r > a$, $H(\xi) = (\sinh\xi/\xi)'$ for $r < a$ (the prime denotes differentiation); ν_+, ν_- are the viscosity of the external fluid and the drop, in accordance with the choice of normal; the function $A_k(\lambda)$ in the terms $z^*(\lambda)$ and $\chi^*_i(r, \lambda)$ must be determined from conditions (26), (28)-(30). We note that because of the integral identity

$$\int_0^\infty r^3S^2(\mu S^2\psi^* - \rho\lambda\psi^*)\,dr + a^4[\partial\{r^{-1}(\mu S^2\psi^* - \rho\lambda\psi^*)\}/\partial r] =$$

$$= \lim_{r\to\infty} r^4\partial\{r^{-1}(\mu S^2\psi^* - \rho\lambda\psi^*)\}/\partial r = 3\rho_+\lambda A_1(\lambda)$$

condition (30) takes on the form $a^3[\rho](g^* - \lambda^2z^*) = 3\rho_+\lambda A_1(\lambda)$.

The general solution of differential equations (28) is written as

$$\chi_i^*(r, \lambda) = E_i^m \frac{H\left(r\sqrt{\lambda/\lambda_m}\right)}{H\left(a\sqrt{\lambda/\lambda_m}\right)} F_m^j C_j(\lambda)$$

($\{\lambda_m\}$ are the roots of the polynomial $\det(\rho\lambda B^{ij} - K^{ij})$, the matrix $\{E^m{}_i\}$ is determined by the equations $(\rho\lambda_m B^{ij} - K^{ij})E^m{}_j = 0$ and $\{F^i{}_m\} = \{E^m{}_i\}^{-1}$). Evidently, such matrices $\{E^m{}_i\}$ and $\{F^i{}_m\}$ exist and the numbers $\{\lambda_m\}$ are positive, since the constant matrices $\{B^{ij}\}$ and $\{K^{ij}\}$ are symmetric and positive definite. Of course, this procedure must be independently carried out for the outer and inner fluids with appropriate choice of functions $H(\xi)$. The functions $C_i(\lambda) = \chi^*_i(a, \lambda)$ are found in (29), which in turn contains $\psi^*(r, \lambda)$. After the requisite computations, we obtain the law for the motion of the center of mass of the drop

$$\lambda^2 m^*(\lambda)z^*(\lambda) = (\rho_0 - \rho)g^*(\lambda). \tag{31}$$

Here

$$m^*(\lambda) = \rho_0 + \frac{\rho}{2} + \frac{9M^*(\lambda)}{2a^2\lambda};$$

$$\frac{1}{M^*(\lambda)} = \frac{1}{\mu X\left(a\sqrt{\lambda/\nu}\right)} + \frac{1}{2\mu + 3\mu_0 X_0\left(a\sqrt{\lambda/\nu_0}\right) + 2Q(\lambda)};$$

$$X(\xi) = 1 + \xi, \quad X_0(\xi) = \frac{(6+\xi^2)\xi - 3(2+\xi^2)\,\mathrm{th}\,\xi}{3[(3+\xi^2)\,\mathrm{th}\,\xi - 3\xi]};$$

$Q(\lambda) = Q_{ij}(\lambda)\sigma^i\sigma^j$; $Q_{ij}(\lambda)$ is the inverse of the positive definite matrix with elements

$$R^{ij}(\lambda) = \lambda a\beta^{ij} + 2a^{-1}\varkappa^{ij} + 2K^{ik}E_k^m Z\left(a\sqrt{\lambda/\lambda_m}\right)F_m^j +$$

$$+ K_0^{ik}E_{0k}^m Z_0\left(a\sqrt{\lambda/\lambda_{0m}}\right)F_{0m}^j,$$

$$Z(\xi) = 1 + \frac{\xi^2}{2(1+\xi)}, \quad Z_0(\xi) = \frac{(2+\xi^2)\,\mathrm{th}\,\xi - 2\xi}{\xi - \mathrm{th}\,\xi}$$

(the index 0 refers to all quantities related to the drop).

Since $X(0) = X_0(0) = Z(0) = Z_0(0) = 1$, then for constant acceleration of external forces g we have the asymptotic velocity of the drop for t → ∞:

$$\frac{dz}{dt} = \frac{2a^2}{9\mu}(\rho_0 - \rho)g\left(\frac{3\mu + 3\mu_0 + 2q}{2\mu + 3\mu_0 + 2q} - \frac{a}{\sqrt{\pi\nu t}} + O\left(t^{-3/2}\right)\right), \tag{32}$$

$$q = Q(0) \text{and } R^{ij}(0) = 2a^{-1}\varkappa^{ij} + 2K^{ij} + K_0^{ij}$$

(in [12], the factor 2 is missing in front of q). Equality (32) for q = 0 (pure boundary separation) contains the well-known Hadamard–Rybzynski formula [1, 2]. With growth in q, the velocity of descent of the drop is retarded, and in the limit q = ∞ (32) is transformed into the Stokes formula [7]. This is the limit obtained for $\mu_0 \to \infty$; therefore accounting for the capillary forces has the effect of increasing the viscosity of the drop μ_0 (see the detailed discussion in [3]).

In the nonstationary case, the appearance of the term $2Q(\lambda)$ in (31) leads to growth of $M^*(\lambda)$, which reduces the rate of acceleration of the drop regardless of the dependence on the sign of σ^i. It is this latter which practically indicates the validity of the Le Chatelier principle in the most general case. Of course, this assertion is based on the positivity of the quadratic form $Q(\lambda)$, which in turn stems from the fundamental principles of thermodynamics: maximum entropy for a closed system and minimum entropy production for an open system close to equilibrium (the Onsager principle is a particular variant [18]).

We note that (31) can be written in the form of Newton's law

$$m*(d^2\mathbf{z}/dt^2) = (\rho_0 - \rho)\mathbf{g}, \tag{33}$$

where the effective "mass" m(t) plays the role of convolution operator with symbol $m^*(\lambda)$:

$$\left(m*\frac{d^2\mathbf{z}}{dt^2}(t)\right) = \left(\rho_0 + \frac{\rho}{2}\right)\frac{d^2\mathbf{z}}{dt^2}(t) + \frac{9}{2a^2}\int\limits_0^t M(t-\xi)\,d\mathbf{z}(\xi)$$

($M(t)$ has the Laplace transform $M^*(\lambda)$). Moreover, in (33) it is possible to consider that $\mathbf{g}(t)$ has variable direction, since the superposition principle is valid for linear equations [13].

We assume that at the initial moment, there is a weakly linear distribution of intensive parameters $\tau_i = \mathbf{w}_i \cdot \mathbf{x}$. After similar calculations, we obtain the law of motion of the center of mass of the drop

$$m*(d^2\mathbf{z}/dt^2) = (\rho_0 - \rho)\mathbf{g} + \mathbf{f}_\sigma, \tag{34}$$

where the capillary force $\mathbf{f}_\sigma(t)$ has the Laplace transform

$$\mathbf{f}_\sigma^*(\lambda) = -3a^{-1}L^*(\lambda)\sigma^i\mathbf{W}_i^*(\lambda),$$

$$L^*(\lambda) = \frac{\mu X(a\sqrt{\lambda/\nu})}{2\mu + \mu X(a\sqrt{\lambda/\nu}) + 3\mu_0 X_0(a\sqrt{\lambda/\nu_0}) + 2Q(\lambda)},$$

$$\mathbf{W}_i^*(\lambda) = \mathbf{w}_i^*(\lambda) + Q_{ij}(\lambda)\left(K^{jk} - K_0^{jk}\right)\mathbf{w}_k^*(\lambda).$$

It follows from the properties of solutions of linear parabolic equations that $\nabla\tau_i$ at infinity will always coincide with $\mathbf{w}_i$, and therefore $\mathbf{w}^*_i(\lambda) = \mathbf{w}_i/\lambda$. Evidently, to a first approximation, formula (33) can be used as a closed form, phenomenological model of the motion of an emulsion by identifying the vector $\mathbf{w}_i$ with the local gradient $\nabla\tau_i$ at the point where the drop is located, which changes with time and has Laplace transform $\mathbf{w}^*_i(\lambda)$. In the one-dimensional case $\tau = \theta$, (34) is an exact result [11], where the work of dilatation of the surface Γ is not taken into account in the energy equation. In conclusion, note that the capillary forces tend to minimize the total Gibbs themodynamic potential, which draws the drop to regions of smaller values of surface tension σ.

LITERATURE CITED

1. J. Hadamard, "Mouvement permanent lent d'une sphere liquide et visqueuse dans un liquide visqueux," C. R. Acad. Sci., 152, No. 25 (1911).
2. W. Rybczynski, "Uber die fortschreitende Bewegung einer flussigen Kudel in einem zahen Medium," Bull. Int. Acad. Polon. Sci. Cracovia. Ser. A, No. 1 (1911).
3. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Fizmatgiz, Moscow (1959).
4. F. Sy and E. N. Lightfoot, "Transient creeping flow around fluid spheres," Am. Inst. Chem. Eng. Jour., 17, No. 1 (1971).
5. A. S. Povitskii and L. Ya. Lyubin, Fundamentals of the Dynamics and Heat and Mass Exchange of Fluids and Gases in Conditions of Weightlessness [in Russian], Mashinostroenie, Moscow (1972).
6. Yu. K. Bratukhin, "Thermocapillary drift of droplets of a viscous fluid," Izv. Akad. Nauk SSSR, Mekh. Zhid. Gaza, No. 5 (1975).
7. J. Happel and G. Brenner, Hydrodynamics at Small Reynolds Numbers [Russian translation], Mir, Moscow (1976).
8. S. S. Dukhin, "Dynamic adsorption layer and the Marangoni–Gibbs effect," in: Contem-

porary Theories of Capillarity: Towards 100 Years of the Gibbs Capillary Theory [in Russian], A. I. Rusanov, and F. Ch. Gudrich, eds., Khimiya, Leningrad (1980).
9. A. L. Gonor and V. Ya. Rivkind, "Dynamics of drops," in: Science and Technology Summaries, Mechanics of Gases and Fluids Series [in Russian], Vol. 17, VINITI, Moscow (1982).
10. Yu. P. Gupalo, A. D. Polyanin, and Yu. S. Ryazantsev, Mass and Heat Exchange for Reacting Particles with Flux [in Russian], Nauka, Moscow (1985).
11. L. K. Antanovskii and B. K. Kopbosynov, "Nonstationary thermocapillary drift of drops of a viscous fluid," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1986).
12. L. K. Antanovskii, "Effect of capillary forces on nonstationary motion of a drop in a uniform fluid," in Hydromechanics and Heat and Mass Exchange in Conditions of Weightlessness [in Russian], Akad. Nauk SSSR Sib. Otd., Novosibirsk (1988).
13. A. E. Rednikov and Yu. S. Ryazantsev, "Nonstationary motion of a drop under the action of capillary and volume forces," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1991).
14. V. G. Babskii, N. D. Kopachevskii, A. D. Myshkis, et al., The Hydromechanics of Weightlessness [in Russian], Nauka, Moscow (1976).
15. L. K. Antanovskii, "Symmetrization of the equations of capillary fluid dynamics," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1990).
16. L. G. Napolitano, "Thermodynamics and dynamics of pure interfaces," Acta Astronautica, 5, No. 9 (1978).
17. J. W. Gibbs, Thermodynamics, Statistical Mechanics [Russian translation], Nauka, Moscow (1982).
18. S. DeGroot and P. Mazur, Non-Equilibrium Thermodynamics, Am. Elsevier, New York (1962).

THE KOENIG FORCE IN A COMPRESSIBLE FLUID

A. A. Doinikov and S. T. Zavtrak

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In publications referring to the Koenig force (see, for example [1-3]), it is assumed that the acoustic wavelength is much larger than the separation between the dispersed particles. Such an assumption allows fluid compressibility to be neglected, but it is valid only for low frequency waves. On the other hand, in practice, for instance in ultrasound technology, radiation of quite high frequency (10^4-10^9 Hz [4]) must be considered. The wavelength of such radiation can be comparable to or even smaller than the separation between particles while remaining many times larger than their dimension. Obviously the neglect of fluid compressibility is then unjustified. The question arises: how does the structure of the Koenig force change when fluid compressibility is taken into account? This paper gives an answer to the question.

Thus we need to compute the force of radiative interaction (the Koenig force) of two rigid spherical particles whose centers execute small oscillations of circular frequency ω when the separation ℓ between the particles is comparable to the acoustic wavelength $\lambda = 2\pi c\omega^{-1}$. The speed of sound in the fluid is c, and the particles have radii R_1 and R_2.

We examine the issue of small parameters. First, we assume that two standard conditions are satisfied: the fluid vibration is potential, that is, $\mathbf{v} = \nabla\phi$ (ϕ is the potential of the fluid velocity $\mathbf{v}$); and $|\mathbf{v}|/c \ll 1$. The latter condition is indicative of the small amplitude of the wave field. Second, in the solution to the analogous problem for an incompressible fluid, two other small parameters are used: $kR_{1,2} \ll 1$ and $k\ell \ll 1$ ($k = \omega/c$ is the wavenumber), with $kR_{1,2} \ll k\ell$. Their small magnitude and the relation between them follows from the assumption $R_{1,2} \ll \ell \ll \lambda$. Relaxing the requirement $\ell \ll \lambda$ means that only one small parameter, $kR_{1,2}$, remains in which to carry out all expansions.

It is well known that radiation forces, including the Koenig force, are quadratic in the field. Considering this, the problem can be formulated thus: we must find the leading

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